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Double-Diffusive Convection: A report on an Engineering Foundation Conference

By C. F. CHEN
Department of Aerospace & Mechanical Engineering, University of Arizona, Tucson, AZ 85721

AND D. H. JOHNSON
Solar Thermal Research Branch, Solar Energy Research Institute, Golden, CO 80401

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Under the auspices of the Engineering Foundation, financial support of the National Science Foundation, and the cochairmanship of the authors, a conference on 'Double-Diffusive Convection' was held from 14–18 March 1983 in Santa Barbara. The conference attracted more than seventy scientists and engineers working in various disciplines, and 45 talks were presented. There was an *ad hoc* film session which ended with a demonstration of a laboratory experiment on crystallization in a double-diffusive system.

1. Introduction

In a recent review, Huppert & Turner (1981*a*) noted that the interactions between fluid-dynamicists who are knowledgeable about double-diffusive effects and scientists and engineers working in other fields have been mostly fortuitous. The purpose of this conference was to provide an opportunity for more systematic interactions between these disciplines. There were nine organized paper sessions, each devoted to one discipline, an opening session, and a closing panel discussion. The themes of these sessions, and their organizers, were:

(i) Introduction and opening panel discussion;
(ii) Theory (Kelly, UCLA);
(iii) Solar Ponds (Zangrando, Solar Energy Research Institute);
(iv) Chemistry (Miller, Livermore);
(v) Astrophysics (Knobloch, UC Berkeley);
(vi) Oceanography (DeLisi, Science Applications, Inc.);
(vii) Geology (McBirney, Oregon);
(viii) Metallurgy (Bertram, Sandia);
(ix) Geophysics (Busse, UCLA);
(x) Laboratory Experiments (Incropera, Purdue);
(xi) Panel discussion (session leaders).

In addition to the regularly scheduled sessions, there was one supplementary paper session and a film session with a laboratory demonstration. A preliminary report of the conference was presented by Huppert (1983).

In the opening session, Turner (ANU) gave a succinct review of the historical development of the field, amply illustrated with slides and film sequences. In the early years, the focus was on one-dimensional problems in which both the thermal and solutal gradients are in the vertical direction. Convective motions can occur either in the 'finger' regime, in which the more slowly diffusing component is heavy on top, or in the 'diffusive' regime, in which the component with the larger diffusivity is heavy on top. In either case, there is a tendency for horizontal convecting layers to develop. Later, the studies were extended to two-dimensional configurations in which the property gradients are no longer simply vertical. Such situations can be obtained, for example, by lateral heating of a solute gradient, by inserting a sloping plate into a vertically stratified fluid, or by the lateral mixing of different water masses. In these cases too, it is clear that double-diffusive systems develop into convecting, nearly horizontal layers at the slightest provocation. Turner then described his more recent research on the effect of crystallization in double-diffusive systems. Crystallization may be produced by cooling the boundary of a container, or by bringing into contact two bodies of solution with different temperature and solute compositions. The density change in the solution due to the crystallization process plays an important role in the subsequent behaviour of the system (Turner & Gustafson 1981; Huppert & Turner 1981*b*).

2. Theory

Huppert (Cambridge) commenced the discussion of theoretical aspects of double-diffusive convection with a survey lecture. The stability problem for a horizontal layer of homogeneous fluid heated from below serves as basis for stability considerations of one-dimensional double-diffusive systems, either in the 'finger' mode or the 'diffusive' mode, and the theory has been extended to include cross-diffusion terms and finite-amplitude effects. There is a limit to the length of fingers for a given buoyancy flux; when this is exceeded, they break down to form a convecting layer. The criterion for such collective instability of fingers was first proposed by Stern (1969) and more recently refined by Holyer (1981). In the 'diffusive' case, as exemplified by the heating of a salinity gradient from below, a linear stability analysis predicts the onset of overstability (Baines & Gill 1969). Veronis (1965, 1968), using a severely truncated Fourier-series representation, found that, for finite-amplitude disturbances, steady convection may occur at thermal Rayleigh numbers less than the critical value predicted by linear theory. Huppert & Moore (1976) used a numerical method to investigate the full two-dimensional equations for a fluid system with various values of τ and σ ($\tau = \kappa_S/\kappa_T$, where κ_S and κ_T are the diffusivities of the slower- and faster-diffusing components respectively, and $\sigma = \nu/\kappa_T$, where ν is the kinematic viscosity). They found that nonlinear steady convection could occur with parameter values for which no linear oscillatory motion is possible.

There followed two presentations on the nonlinear stability aspects of the diffusive problem. (The 'finger' instability, being a straightforward steady convection, does not yield such a rich variety of results.) Proctor (Cambridge) reported on the results of a perturbation analysis for the case $\tau \ll 1$ (Proctor 1981). Steady subcritical convection is possible for any given initial salt gradient, provided that τ is sufficiently small and the thermal Rayleigh number is beyond the critical value. Further, analytical solutions exist (Knobloch & Proctor 1981) in the limit when the Rayleigh number for the onset of overstable oscillations and that for the onset of steady convection are close together. In this case, the leading term describing the time evolution of oscillation amplitude satisfies an equation of a nonlinear oscillator. The steady branch was found to be always subcritical. Bretherton (MIT) considered a horizontally unbounded layer. Numerical simulations show that nonlinear interactions of wave packets may develop into chaos near the critical condition for the onset of oscillating instability. In addition, for convective motion in two-dimensional rolls, the simulation results show period-doubling behaviour and transition to chaos.

A two-dimensional configuration was discussed by Chen (Arizona); namely a fluid with vertical salinity gradient confined within a narrow slot whose two walls are held at different temperatures. The slot can be set at any angle to the vertical, thus generalizing the vertical problem considered by Thorpe, Hutt & Soulsby (1969) and Hart (1971). Linear stability theory predicts the observed critical Rayleigh number and wavelength of the steady convection and the fact that there is no overstability (Paliwal & Chen 1980). However, the motion consists of convection rolls with alternating directions of rotation. Experimental evidence indicates that all convection rolls have the same sense of rotation, rising along the hot wall and descending along the cold wall. A nonlinear treatment of the problem (Thangam, Zebib & Chen 1982) revealed that the cells with the wrong sense of rotation (descending along the hot wall) are quickly squeezed into interfaces between cells with the correct sense of rotation.

Platten (Mons, Belgium) discussed the Soret effect on double-diffusive convection, reviewing experiments in the light of theoretical results. The heating from below of a solute gradient is complicated by the fact that heat flux depends on the concentration gradient as well as the temperature gradient. As a result, the heat transfer rate exhibits hysteresis effects as the thermal Rayleigh number is increased and then decreased. Castillo (UNED, Spain) presented the results of an energy analysis of the stability of a double-diffusive system with surface-tension effects, in which the free surface was permitted to deform (Castillo & Verlarde 1982). A sufficient condition for stability was established and regions of possible subcritical instability were delineated.

3. Solar ponds

Solar ponds are the main large-scale engineering projects in which double-diffusive concepts are carried into practice. Ironically, the degree of success in the design and operation of such ponds is measured by the absence of convection in the insulating gradient layer. In the actual operation of a pond, two problems are prominent. One is that the salt gradient is not uniform throughout the depth, the other is the long-time behaviour of the gradient layers.

The problem of onset of instabilities in a fluid layer with nonuniform salt gradient was investigated by Bertram (Sandia) using a numerical method, and by Walton (Imperial College) using a perturbation method (Walton 1982). The results show that the instability occurs in the overstable mode in a thin layer at the depth where the salinity gradient is minimum. These and many other features predicated by theoretical considerations show good agreement with observations made by Zangrando (SERI), which motivated the theoretical work. The behaviour of a diffusive interface at large density ratios R_ρ ($R_\rho = \beta\Delta S/\alpha\Delta T$, where β and α are the expansion coefficients for salt S and temperature T respectively), which is the condition in solar ponds, was investigated experimentally and described by Newell (Illinois). The variation of heat flux with R_ρ shows a trend between the constant value proposed by Marmorino & Caldwell (1976) and the sharply decreasing one proposed by Huppert (1971) fitting the data points of Turner (1965). A simple theory based on core-dominated transport was proposed to explain the observed data. Estimated values of the heat flux remained greater than zero up to $R_\rho = 28$.

Jones (Los Alamos) reported the development of a numerical model incorporating the measured flux data to predict the time evolution of the interface between the convecting layer and the non-convecting gradient layer. The results are in reasonable agreement with data from a laboratory experiment.

Also included in this session was a presentation by Narasawa (Northeastern) on the problems of double-diffusive convection in large LNG (liquified natural gas) storage tanks. Because these tanks are likely to receive LNG from a variety of sources with different chemical compositions, multidiffusive convection is likely to occur. These convective motions may develop into the phenomenon known as 'rollover', in which there is a sudden overturning, leading to an increase in the rate of boil-off of gas, with disastrous consequences.

In a comment from the floor, Kantha (Dynalysis) pointed out that double diffusion in porous media has been recognized as playing an important role in groundwater pollution (Griffiths 1981; Nield 1968; Taunton, Lightfoot & Green 1972; Tyvand 1980), and may be an important consideration in heat lost to the ground below unlined solar ponds.

4. Chemistry

In determining transport coefficients in multicomponent fluids, the physical chemists are often faced with unwanted convective effects with double-diffusive origin. Tyrrell (Chelsea College) discussed various facets of this problem. In certain situations, however, double-diffusive effects may be used to determine the transport coefficient in question. This was done by Caldwell (1973) to determine the Soret coefficient of sodium chloride solution.

Preston & Comper (Monash) presented a fascinating paper on convection in aqueous systems containing polymers. A layer of aqueous solution of dextran with the addition of a small amount of PVP is first placed in a container. On top of this layer is introduced a layer of less concentrated dextran solution (Preston *et al.* 1980). Convection cells, similar in shape to salt fingers, are seen to develop at the interface. This has been beautifully illustrated by time-lapse photography. Wells (Uppsala) in the supplementary session, reported finding such finger-like convection in a number of combinations of macromolecular solutions. These systems cannot be explained in the terms generally used to describe the fingering instability mechanism because in this case both the faster- and slower-diffusing components are heavy on the bottom. McDougall & Turner (1982) and McDougall (1983) have discussed the possibility of fingering instability in such a situation when the effects of cross-diffusion coefficients are taken into account. However, comparison with experimental results cannot easily be made because these coefficients for the experimental solutions are unknown at present. Perhaps the existence of such interesting observations may provide an impetus for the determination of these coefficients.

Two other presentations in the supplementary session are related to the present topic. Pearlstein & Kelly (UCLA) presented a discussion of the onset of double-diffusive convection in a reacting fluid layer bounded by catalytic walls. It was first shown that there exist motionless states which are stable to perturbations in the reaction and diffusion equations, and which can then be analysed for double-diffusive instabilities. Brand & Steinberg (UC Santa Barbara) reported the results of a similar study in a porous medium.

5. Astrophysics

In stars, in addition to the thermal and compositional gradients, there exist gradients in magnetic fields and/or rotation. Multidiffusion effects may be expected as a result. The phenomena of overstability, subcritical overturning instability, and fingering all have their counterparts in the presence of gradients in angular momentum and magnetic field strength. Weiss (Cambridge) described a numerical model of a case with stabilizing thermal gradients and destabilizing magnetic field, and used the results to explain the presence of sunspots and granules on the surface of the sun. Spruit (Max-Planck-Institut, Munich) discussed the effect of rotation. At high rotation rates, the motion is dominated by baroclinic and shear instabilities. At low rotation rates, however, multicomponent diffusion becomes important. The mixing effect of these instabilities on the chemical composition of the star has important consequences for our understanding of stellar evolution.

In the astrophysical context, Toomre (Colorado) presented results of his numerical model of a salinity gradient being heated from below, extending the calculations of Huppert & Moore (1976) to $\tau = 0.01$ at high thermal Rayleigh number $R_T \sim 10^6$. He also described time-dependent calculations of a series of diffusive layers which showed plumes breaking away from the boundary layers in a highly variable manner. The time-averaged flux ratio was nevertheless comparable with the laboratory values.

Rosner (Harvard) gave a review of magnetic-buoyancy instabilities. Such instabilities may occur in the interior of stars, where they may account for the 'activity' of solar-type stars, and in the interstellar medium, where they may play a role in the formation of stars. If we study a thin layer of fluid in which a weak horizontal one-dimensional magnetic field is embedded and assume that all motions of interest are subsonic, with no differential rotation, and constant properties, then the equations describing the time evolution of the system reduce to exactly the same form as those describing the thermohaline problem where now entropy plays the role of temperature and the magnetic field plays the role of salt. These effects are discussed in Schmitt & Rosner (1983).

6. Oceanography

The branch of science that spurred the study of double-diffusive phenomena, oceanography, remains very active in this area, though there is now much debate on the question of the quantitative importance of these processes in the ocean relative to other mixing mechanisms. Because of the rapid transport of salt and heat (and perhaps nutrients) through the fingers, this instability phenomenon is the most studied by the oceanographers. There were four presentations concerning fingers, three observational and one theoretical, and one paper on horizontal intrusions.

Schmitt (WHOI) reviewed the observational evidence, and concluded that the majority of the mixing events in a frontal interleaving zone are double-diffusive in origin, and only a small fraction are caused by shear instability due to low Richardson number. There is a strong suggestion that double-diffusive processes are the most important mixing processes acting at fronts between water masses. He also presented calculations based on his earlier similarity solution (Schmitt 1979) of characteristics of salt fingers for a Prandtl-number range from 10^{-7} to 10^4 and diffusivity ratios from 1 to 10^8. These results should be applicable to most systems discussed at this conference. Gregg's (Washington) observations indicate strong energy dissipation in the finger regions. He made a plea to laboratory experimentalists to measure flux laws at low density ratios, which is the prevailing situation in the ocean. Gargett's (IOS) measurements of quasi-periodic temperature fluctuations of limited amplitude can be attributed to salt fingers. The buoyancy flux due to temperature shows order-of-magnitude agreement with laboratory results. Stern (URI) considered the salt-finger regime between two deep layers of fluid with different temperature and salinity. By the use of a variational principle advanced by him earlier (Stern 1982), he was able to place an upper bound on the salt flux through the fingers.

The laboratory experiments of Ruddick & Turner (1979) have shown that convection in layers may be generated by horizontal intrusions of different water masses. Holyer (Bristol) has carried out a linear stability analysis for an unbounded, vertically stratified fluid with compensating horizontal temperature and salinity gradients. The results predict the growth rate and vertical scale of the most-unstable perturbations, but it is still difficult to reconcile these with the laboratory results or with the field observations (Toole & Georgi 1981).

7. Geology

Volcanism and magmatic crystallization are two important processes involved in the formation of the Earth's crust. Both processes have the necessary ingredients for double-diffusive convection. Since the early work of Turner & Gustafson (1978) and Chen & Turner (1980), double-diffusive effects in geological systems are under active research by a number of fluid-dynamicists and geologists. The earlier, mainly qualitative, studies are now being replaced by more quantitative work, which is paying more attention to the scaling problems.

McBirney (Oregon) reported on a series of experiments in which an aqueous solution of Na_2CO_3 was crystallized by cooling different sections of a laboratory 'magma chamber' with a peaked roof. The resulting stratification of the solution leads to a convincing explanation of the unmixing of magma as evidenced by the abrupt changes in the composition of volcanic rocks (McBirney 1980). Spera (Princeton) reported the results of boundary-layer calculations using large viscosity variations with temperature when a magma chamber is being cooled at a sidewall (Spera, Yuen & Kirschvink 1982). The results suggest that estimates of boundary-layer thickness, heat flux and shear stress at the wall may differ by an order of magnitude from those predicted assuming the viscosities to be constant. If cooling is strong enough to cause crystallization at the wall, then the effect of the compositional boundary layer must be taken into account. Depending on the chemical composition of the magma, the flow in the compositional boundary layer may be aiding or opposing the flow in the thermal boundary layer. Nilson (SSS) solved the self-similar boundary-layer equations and delineated the different flow regimes according to the regions of the parameter space, which consists of the Prandtl number, the Lewis number and the relative buoyancy. When the flow contains a reversed region, no self-similar solution can exist. Baker (Oregon) pointed out that the interactions between thermal and compositional boundary layers would produce distributions of trace elements very different from the initial well-mixed values. As a result of crystallization and upflow, there should be regions of enrichment and depletion of trace elements in the segregated magma. A study of the trace element distributions as a function of position may indicate whether double-diffusive convection has occurred.

In the supplementary session, Hsui & Riahi (Illinois) presented a stability study of double-diffusive systems with crystallization, and Rice (Florida Institute of Technology) questioned the way in which laboratory results are being used to predict full-scale geological phenomena.

8. Metallurgy

At the solid–liquid interface during crystallization, concentration and thermal gradients may cause double-diffusive convection which in turn affect the interfacial stability. Coriell & McFadden (National Bureau of Standards) addressed themselves to this problem. The normal double-diffusive instability problem is complicated by the fact that (i) the solid–liquid interface is moving, (ii) the interface is not constrained in its shape, and (iii) the concentration gradient decays exponentially away from the interface. A numerical procedure has been devised to solve the stability equations and the results differ substantially from those obtained by ignoring double-diffusive effects (Coriell *et al.* 1980).

Adornato, Chang & Brown (MIT) considered the problem of growing semiconductor crystals. They pointed out that the concentration gradient in the melt near the solid–liquid interface may be gravitationally stable or unstable depending on the semiconductor material. A finite-element numerical procedure was used to solve the problem, and the instability characteristics were identified.

Glicksman & Singh (Rensselaer Polytechnic Institute) described careful experiments designed to examine the growth of dendrites. It was found that the rate of growth of the dendrites is increased by the presence of impurities. The growth rate was also found to be a function of the orientation of the dendritic growth axis with respect to gravity.

9. Geophysics

Soward (Newcastle) introduced the audience to the problem of motion in the Earth's core. There the conducting fluid is in rotation, it is permeated with a magnetic field, and is subjected to a thermal gradient. The opportunities for multicomponent convection abound. Instabilities are characterized by inertial waves together with the so-called MAC waves and torsional oscillations.

Peltier (Toronto) discussed the effect of double diffusion in shear-flow instabilities. He found that in convective instabilities generated by superadiabatic sublayers the growth rate is much larger than that due to Kelvin–Helmholtz instability alone. Busse (ULCA) considered baroclinic instabilities with double-diffusive effects. It was pointed out that the results obtained by considering the diffusive effects, both thermal and momentum, and then approaching the limit of zero diffusivities, are different from those obtained by setting the diffusivities to be zero at the outset. Antar (Tennessee) presented results of a three-dimensional linear stability analysis of a baroclinic flow at low Richardson numbers (Antar & Fowlis 1982, 1983).

10. Laboratory experiments

Viskanta (Purdue) gave a comprehensive review of optical methods suitable both for flow visualization and for quantitative measurements in double-diffusive experiments. These techniques have been used with great success to investigate double-diffusive convection (Chen & Turner 1980; Grange, Viskanta & Stevenson 1976; Huppert & Linden 1979; Huppert & Turner 1980; Lewis, Incropera & Viskanta 1982; Paliwal & Chen 1980; Poplawsky, Incropera & Viskanta 1981; Skok & Chen 1974). Tsinober (Tel Aviv) used the dye-trace method to visualize the motion in a plume rising from a heating element in a stratified fluid. He presented a number of beautiful slides showing a flow pattern like a Christmas tree; it consisted of convecting layers sloping downward from the plume as they moved towards the cooler surroundings. Ruddick (Dalhousie) has used the optical rotation property of sugar solution to study the interleaving horizontal intrusions at a vertical interface separating two bodies of fluid stratified with solutes having different diffusivities (Ruddick 1981).

The propagation of solitary waves on diffusive and finger interfaces were investigated by Hurdis (Johns Hopkins). On diffusive interfaces, the solitary wave rapidly attenuates. On finger interfaces, the solitary wave initially amplifies and subsequently disintegrates rapidly. Ostrach (Case Western Reserve) reported on the results of experiments in a long horizontal cell with horizontal gradients of concentration and temperature.

Under the general direction of Herbert Huppert, an experiment was performed with the enthusiastic participation of a large number of the conference attendees. An aqueous solution of KNO_3 with specific gravity 1.33 and temperature approximately 70 °C was poured into a Perspex container. To this was carefully added an aqueous solution of $NaNO_3$ with specific gravity of 1.30 at about 4 °C. Crystallization of KNO_3 occurred both in the interior of the lower layer and at the bottom of the container. As the experiment proceeded, the light KNO_3 solution left behind by the crystallization process mixed uniformly in the lower layer, and eventually the density of the lower layer decreased sufficiently for there to be complete mixing of the two fluids. The experiment models the injection of hot, heavier magma into a relatively cold, lighter homogeneous magma (Huppert & Turner 1981*b*).

This demonstration, and indeed the other laboratory models described at earlier sessions of this conference, made it clear that a large amount of physical understanding can be gained using simple experiments. In the final panel discussion many of the speakers emphasized the need for more carefully controlled measurements, with experiments conducted closer to the parameter ranges for which clear theoretical predictions can be made. Results which can be tested unambiguously using field observations of various kinds will also be important, and there is still much scope for those working in different disciplines to learn from one another.

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